# NONLINEAR CIRCUIT ANALYSIS OF LASER DIODES UNDER MICROWAVE DIRECT MODULATION

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## ABSTRACT

A microwave nonlinear circuit analysis technique which can account for all known steady-state responses has been developed and applied to the large-signal characterisation of directly modulated laser diodes. An equivalent circuit derived from the rate equations is used to model the laser diode. The proposed technique is based on a harmonic balance algorithm which represents two-tone inputs by describing frequencies. Second harmonic and third-order intermodulation distortion results for a GaAlAs diode have been compared with corresponding measured data to validate the approach taken. Aperiodic responses are detected by means of bifurcation theory.

## INTRODUCTION

The considerable interest in directly modulated fibre-optic links is due to the potentially superior loss and bandwidth performance which optical fibre can offer over conventional microwave transmission lines. However, the available bandwidth and distortion levels of a link are ultimately dictated by the laser diode. The requirement for high signal-to-noise ratio (SNR) in an analogue link is usually satisfied by large-signal modulation of the laser diode. When multitone inputs are present, this results in intermodulation distortion which is undesirable in multichannel transmission. Therefore, it is necessary to develop accurate large-signal modelling techniques in order to achieve an optimal trade-off between SNR and linearity.

Previous investigations [1] of harmonic and intermodulation distortion in directly modulated laser diodes were based on small-signal solutions of the rate equations which are not valid for large-signal inputs. A large-signal equivalent circuit model was analysed in the time domain [2], although the effects of parasitic elements were neglected. The object of this paper is to describe the application of harmonic balance techniques (in conjunction with bifurcation theory) to the large-signal characterisation of directly modulated laser diodes.

## LARGE-SIGNAL MODEL OF LASER DIODE

The dynamic behaviour of photon and electron densities within the diode cavity is described by a pair of single-mode rate equations. These are given by:

$$\frac{dN}{dt} = \frac{I_1}{\alpha} - \frac{N}{\tau_n} - \gamma(N - N_o)(1 - \varepsilon S)S \tag{1}$$

$$\frac{dS}{dt} = \Gamma\gamma(N - N_o)(1 - \varepsilon S)S - \frac{S}{\tau_p} + \Gamma\beta\frac{N}{\tau_n} \tag{2}$$

where $S$ and $N$ are photon and electron densities, $\tau_p$ and $\tau_n$ are photon and electron lifetimes, $\alpha$ is the product of the active region volume with the electron charge, $\Gamma$ is the mode confinement factor, $\varepsilon$ is a gain compression parameter, $\gamma$ is the optical gain coefficient, $N_o$ is the value of $N$ for transparency, $\beta$ is the fraction of spontaneous emission entering the lasing mode and $I_1$ is the current entering the active region.

The terminal voltage of the active region ($V$) is related to the electron density by a classical Shockley expression. An equivalent circuit may then be derived from (1) and (2) in the manner described by Tucker[3], and this can be cascaded with parasitic elements, bias networks and matching networks as shown in Fig.1. The nonlinear resistor $\overline{R}$ models spontaneous recombination, and photon density is modelled as the output voltage. The current sources $\beta I_{sp}$ and $I_{st}$ represent spontaneous and stimulated emission respectively.

It is relatively straightforward to modify the model for multimode operation, or to account for the effect of coherent reflected light from the laser facet. The latter phenomenon is modelled by a light output dependent FM current source ($I_{FM}$) in parallel with the $\beta I_{sp}$ source.

## NONLINEAR CIRCUIT ANALYSIS PROCEDURE

The feasibility of using time domain methods to determine nonlinear distortion effects in laser diodes has been demonstrated [2,4]. It is more convenient, however, to analyse distributed matching elements and complex parasitic networks in the frequency domain. This may be accomplished by using harmonic balance techniques [5,6,7]. However, the *a priori* assumption of periodic steady-state solutions is not always justified. The rate equations describe a second-order nonautonomous system which can exhibit period doubling and chaos [8] for certain parameter values. A large-signal analysis technique should therefore take the possible existence of subharmonics and chaos into account. In the present procedure, a perturbation analysis (based on Melnikov's method [9]) is used to determine the regions of chaos. An example of a bifurcation surface for a Hitachi HLP-3400 diode is shown in Fig.2; operating points selected in the "indentation" may lead to aperiodic responses.



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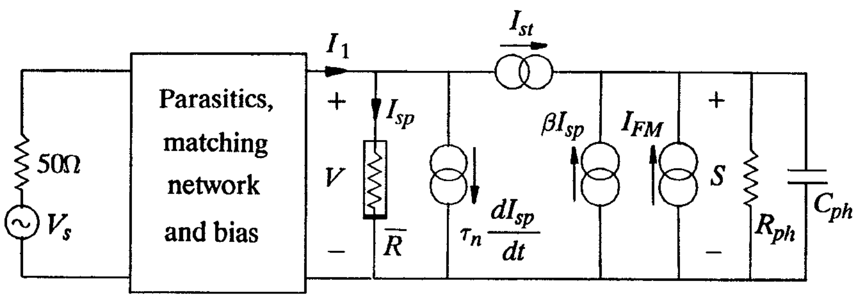


**Fig.1 Large-signal equivalent circuit model of laser diode**

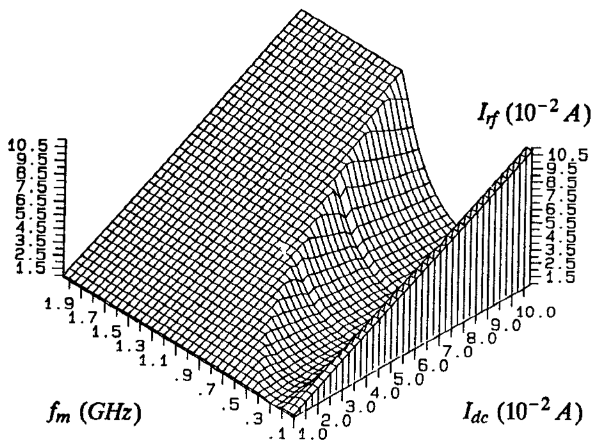


**Fig.2 Bifurcation surface for the Hitachi HLP-3400 diode. $I_{dc}$ is the bias current, $I_{rf}$ the modulation current, and $f_m$ the modulation frequency ($I_{rf} \leq I_{dc}$).**

Having selected a periodic operating point, it is possible to solve the equivalent circuit (Fig.1) by the harmonic balance method. The new algorithm partitions the linear and nonlinear subnetworks into circuits driven by substitution voltages as shown in Fig.3. The substitution voltages $V$ and $S$ are represented by truncated Fourier series. (In this context $S$ represents the photon density rather than a physically meaningful voltage). From an initial estimate of $V$ and $S$, the linear and nonlinear subcircuit currents are calculated using frequency and time domain analyses respectively. The residual error $\varepsilon_r$ is given by the difference between the linear and nonlinear currents at each particular port, and should be zero to satisfy Kirchhoff's laws.

The algorithm seeks to calculate the perturbed substitution voltages $V + \Delta V$ and $S + \Delta S$ which will reduce $\varepsilon_r$ to zero. The voltages $V + \Delta V$ and $S + \Delta S$ are used to obtain truncated Taylor series expansions of the nonlinear subnetwork currents. The nonlinear current terms contributing to the error (i.e. which are a function of $V$ and $S$ alone) can then be extracted from the rest of the series. By the superposition principle [10], the linear subnetwork currents due to $\Delta V$ and $\Delta S$ may also be isolated from those current terms (due to $V$ and $S$) which contribute to the error. In this manner, a sensitivity circuit may be derived (Fig.4). Solving this circuit will yield the perturbation voltages ($\Delta V$, $\Delta S$) required for zero error ($\varepsilon_r = 0$).

However, the truncation of the Taylor series prevents the reduction of $\varepsilon_r$ to zero in one iteration. The variation in the substituted voltages at a given iteration is then calculated by solving the sensitivity circuit driven by the residual error of the previous iteration.

When a two-tone input is present, the Fourier analysis is performed in a one-dimensional frequency domain which is related to the original grid of intermodulation components by a bilinear mapping law [11]. The commensurate excitation frequencies $\omega_1$ and $\omega_2$ are transformed to a pair of describing frequencies which are rationally independent with respect to the order of nonlinearity, $M$:

$$\omega_1^d = M\omega_o \tag{3}$$

$$\omega_2^d = (M + 1)\omega_o \tag{4}$$

where $\omega_o$ is an arbitrary frequency basis, and $M$ corresponds to the maximum order of intermodulation products being considered. As a result, the frequency independence of the Fourier coefficients of the transformed spectrum is guaranteed. Consequently, the original sparse spectrum is replaced by a describing frequency spectrum with $(M + 1)M$ components and no gaps.

## RESULTS

The above algorithm has been used to examine the harmonic and intermodulation distortions of a high-speed GaAlAs single-mode laser diode (Ortel SL-620). There is good agreement between the calculated and measured results [2] as indicated in Fig.5 and Fig.6.

Fig. 5 shows the power ratio of the second harmonic to the fundamental ($P_{2f}/P_f$) as a function of bias current for an input frequency of 2GHz. The threshold current ($I_{th}$) of this device is 21mA. Fig.6 shows the power ratio of the third-order intermodulation products to the carrier ($P_{IM3}/P_f$) as a function of bias current. Equal inputs of -1dBm at 4.00GHz and 4.04GHz were used. In general, there is an improvement in linearity with increasing bias current.

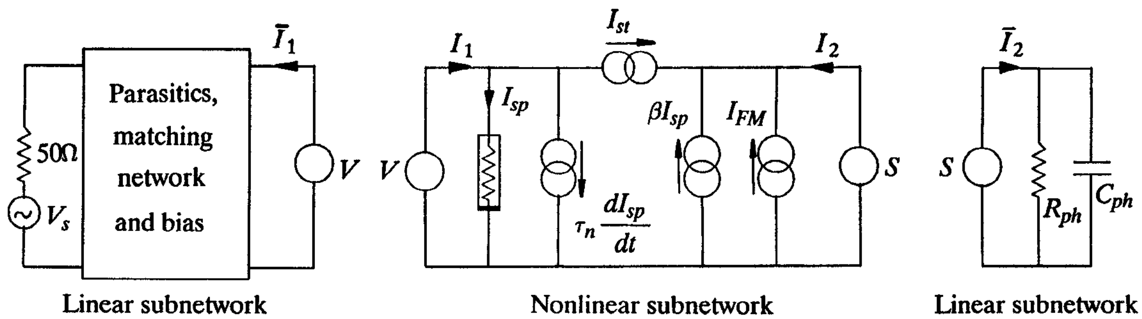


**Fig.3 The partitioned circuit with substitution voltages**

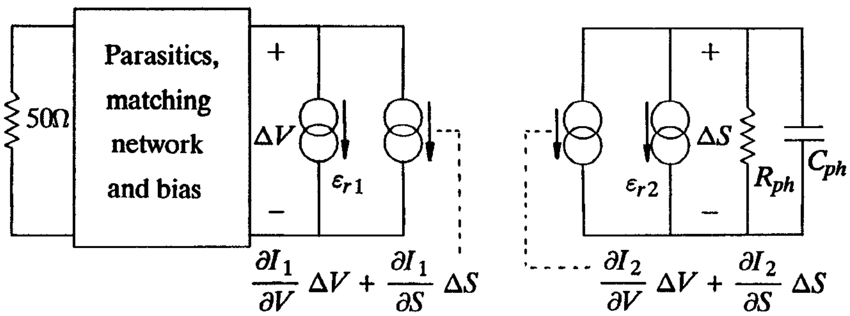


**Fig.4 Sensitivity circuit**

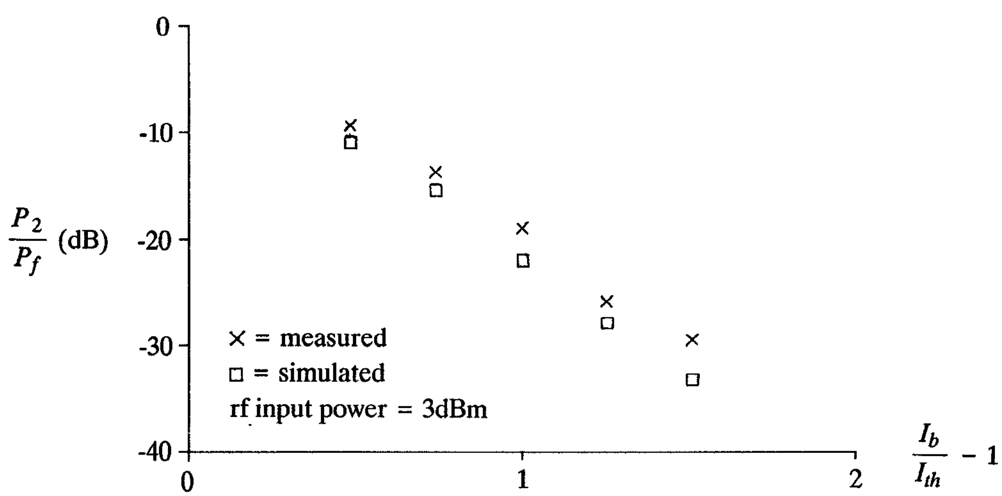


**Fig.5 Power ratio of second harmonic to fundamental as a function of bias current**

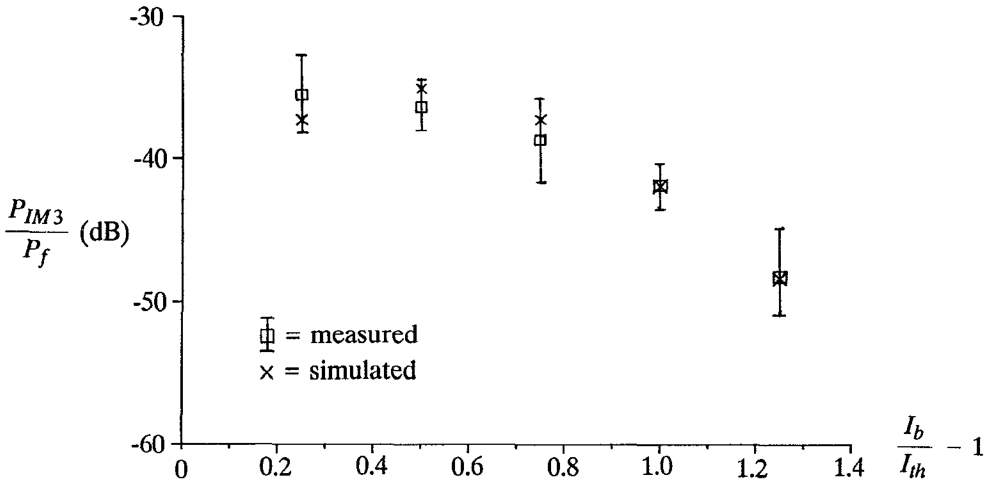


**Fig.6 Power ratio of third-order intermodulation products to carrier as a function of bias current.**

### CONCLUSIONS

The nonlinear distortions of a directly modulated single-mode laser diode have been examined using a new algorithm based on harmonic balance and bifurcation theory. The approach allows for the possibility of chaotic solutions to the circuit equations, unlike other microwave nonlinear analysis techniques [7]. For periodic responses, the nonlinear circuit is solved using a harmonic balance method. Two-tone inputs are handled by describing frequencies, which greatly reduce the computational effort involved in time to frequency domain conversion. Simulated results are shown to agree well with measured data, and indicate the capability of using this approach for the CAD of microwave fibre-optic transmitters.

**Acknowle‹›ments**

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**References**

1 K.Y. Lau and A. Yariv,"Intermodulation distortion in a directly modulated semiconductor injection laser," *Appl. Phys. Lett.*, vol.45, pp.1034-1036, 1984.

2 W.I. Way,"Large signal nonlinear distortion prediction for a single-mode laser diode under microwave intensity modulation," *J. Lightwave Technol.*, vol. LT-5, pp.305-315, 1987.

3 R. S. Tucker,"Large-signal circuit model for simulation of injection-laser modulation dynamics," *Proc. Inst. Elec. Eng.*, vol. 128, pt. I, pp. 180-184, 1981. pp. 985-991, 1987.

4 P.A. Morton, R.F. Ormondroyd, J.E. Bowers and M.S. Demokan,"Large-signal harmonic and intermodulation distortions in wide-bandwidth GaInAsP Semiconductor Lasers," *IEEE J. Quantum Electron.*, vol. QE-25, pp.1559-1567, 1989.

5 S. Iezekiel, C.M. Snowden and M.J. Howes,"Harmonic balance model of laser diode," *Electron. Lett.*, vol. 25, pp.529-530, 1989.

6 S. Iezekiel, C.M. Snowden and M.J. Howes,"Large signal analysis of laser diodes for microwave fibre-optic links," *Proc. 19th European Microwave Conf.* (London), pp.985-990, 1989.

7 V. Rizzoli and A. Neri,"State of the art and present trends in nonlinear microwave CAD techniques," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-36, pp.343-365, 1988.

8 C.H. Lee, T.H. Yoon and S.Y. Shin," Period doubling and chaos in a directly modulated laser diode," *Appl. Phys. Lett.*, vol. 46, pp.95-97, 1985.

9 J. Guckenheimer, "Toolkit for nonlinear dynamics," *IEEE Trans. Circuits Sys.*, vol. CAS-30, pp.586-590, 1983.

10 S.A. Maas, *Nonlinear Microwave Circuits,* Norwood,MA: Artech House,1988.

11 D. Hente and R.H. Jansen, "Frequency Domain Continuation Method for the Analysis and Stability Investigation of Nonlinear Microwave Circuits", IEE Proceedings, vol. 133, pt. H, pp. 351-362, 1986.